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Energy Nature of Centrifugal and Newtonian Forces

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Abstract: Newton's forces are part of the fundamental law of dynamics, and the centrifugal forces determine the increment of the kinetic energy of a rigid body on the incremented relevant kinematic parameters. For a Newtonian force, it is the change of position of the center of mass of the body in relation to the coordinate system of the observer, for the centrifugal forces—the change in the distance between the center of mass and the instantaneous center of velocity. As the second case does not require coordinate system of the observer, the centrifugal force should be considered more objective and advantageous; for example, when searching for dynamic balance mechanisms through the coordinated movement of the center of mass and the instantaneous center of velocity.

Keywords: Generalized forces and coordinate, kinetic energy, Newton's and centrifugal forces.

1. Introduction

Centrifugal forces are forces in non-inertial reference systems, which are not depended from the velocity of motion of material point in these systems and (linear and angular) accelerations of these systems relative to a local inertial system [1].

The interest to centrifugal forces is explained by their important role in solving fundamental and applied issues of mechanics. Fabulousness and reality issues, extensively discussed on the internet and other publications such as Ref. [2], are related with incomprehension of force in classical mechanics, which is introduced by Newton axioms as some abstract causes of motion. Such idea of principal concept of mechanics does not always lead to clear and sometimes to conflicting judgments [3, 4].

Firstly, it is connected with the restrictions placing on inertial systems. Newton's laws are applied for them because homogeneity and isotropy of space and time follow from the first law of motion. The contradictions of classical mechanics were described in Ref. [4]. It was noted that some motions, which allows for classical mechanics, cannot be implemented in reality.

Commitment to Newton's laws in non-inertial systems, moving with the acceleration or rotating about inertial systems, necessitates to superinduce "inertial forces", which do not have exact definition as Newtonians. They allow finding accelerations from the force balance. But for determining of motion equation, it needs to lay down conditions of forces transformation and accelerations, moving from inertial system to non-inertial system, additionally [5].

Above-mentioned and other difficulties of classical mechanics made it possible to building another system, in which force concept was not used as undefined. Energy models mechanics were considered preferably. In Ref. [6], Hertz considered only kinetic energy. Helmholtz model [7] took into account the potential energy of body too.

In Ref. [4], it is noted that the energetic model, with the substitution of abstract concepts of "force" and "mass" for energetic (kinetic and potential energy), allows clarifying and getting more information about motion than following from fundamentals of classical mechanics.

2. Assumptions and Key Ratios

In contrast to Ref. [6], we understand energy as a generalized scalar function of different types of motion [2, 8]. It is natural to assume that arguments of this function must be scalar invariants of the equations of motion. In accordance with the basic postulate of mechanics, behavior of the system depends on the position of the particles and their velocity. Therefore, in the general case of motion of a continuous with random properties as parameters generalized scalar functions must be 12 invariants, including modules of the vectors of displacement and speed, the path (the integral of the modulus of the velocity with respect to time), three invariant of tensors of strain and strain rate, and three integral option at the time of the three invariants of the strain rate tensor, including the parameter of hardening Odkvist [1]. To describe the laws of motion of solid bodies, it is necessary to consider the first three of the above invariants.

Modern views do not contradict the assumption that the energy E can be represented as a sum of terms $E_i(\xi_i)$ [8]:

$$E = \sum_i E_i(\xi_i)$$

all of which depends only on one invariant ξ_i and characterizes one of the forms of energy $E_i = k_i\xi_i$. Here, k_i—scalar coefficient characterizing the physical properties of moving material substance or the environment in which there is a movement. In particular, when rigid body moving displacement vector characterizes the change in potential energy, for example, in the terrestrial gravitational field, the velocity vector—the kinetic energy, the third invariant (path) may account for the dissipative processes. Furthermore, for isolated systems, energy of external influences must be taken into account [8].

Considering the interaction of solids as an exchange of energy, the behavior of the mechanical system is determined by the law of conservation of energy in the following form:

$$dE = \sum_i dE_i(\xi_i) = 0$$

The increment of energy dEi can be represented via generalized forces Qij and the generalized coordinates q_j:

$$dE_i = \sum_j \frac{\partial E_i}{\partial q_j} dq_j = \sum_j Q_{ij} dq_j$$
$$Q_{ij}(q_j, t) = \frac{\partial E_i}{\partial q_j} \quad (1)$$

which plays a decisive role in calculating the changes of different types of energy E_i.

Since the choice of generalized coordinates has several variants (for example, linear and angular position or velocity in Cartesian or polar coordinates), the change of each of the types of energy can be characterized by various combinations of generalized forces Q_{ij}, corresponding to various combinations used to describe the motion of generalized kinematic coordinate q_j. But in any form, these combinations should yield the same result for the increment of energy dE_i. Only the overall change dE_i is invariant based on the sum of all the terms $Q_{ij}dq_j$.

In accordance with the definition in Eq. (1), each generalized force Q_{ij} should be classified by the type of energy E_i, the increment of which it determines and the type of kinematic coordinates q_j, the increment of which to be used for calculating the corresponding increments of the energy dE_i.

To clarify the differences between Newtonian and centrifugal forces, possible ways are considered to determine the kinetic energy E_k at plane-parallel movement of solid body [8, 9]. In general, for a solid body of mass m, we have the integral

$$E_k = 0.5\int_m v^2 dm$$

Taking into account the kinematic relations:

$$v_x = (v_x)_P - \omega(y - y_P)$$
$$v_y = (v_y)_P + \omega(x - x_P) \quad (2)$$

where ω—angular velocity of rotation of the body, $(v_x)_P$, $(v_y)_P$, v_x, v_y—components velocity of the particle at the pole $P(x_P, y_P)$ and at the point with coordinates x, y, after integration we obtain [8, 9]

$$E_k = 0.5mv_P^2 + 0.5\omega^2 J_P - \omega m[(v_x)_P(y_C - y_P) - (v_y)_P(x_C - x_P)] \quad (3)$$

where x_C, y_C—the coordinates of the center of mass of the body, J_P—moment of inertia of the body relative to the axis passing through the pole P,

$$J_P = \int_m [(x-x_P)^2 + (y-y_P)^2]dm$$

To identify the energy nature of centrifugal and Newton's forces, two options when pole P is aligned with the center of mass C and the instantaneous center of velocity (ICV) are considered.

$$E_k = 0.5mv_C^2 + 0.5\omega^2 J_C = 0.5\omega^2 J_{ICV} \quad (4)$$

where J_C, J_{ICV}—moments of inertia of the body relative to the axis passing through the center of mass C and ICV:

$$J_C = \int_m \rho^2 dm$$

$$J_{ICV} = J_C + m(CP)^2$$

In the last equation, CP—distance between the center of mass C and ICV. Coordinates ICV are found from Eq. (2), subject $(v_x)_{ICV} = 0$, $(v_y)_{ICV} = 0$,

$$x_{ICV} = x_P - (v_y)_P / \omega$$

$$y_{ICV} = y_P + (v_x)_P / \omega \quad (5)$$

To understand the energy meaning of Newtonian forces, the case when the pole P is aligned with the center of mass C is considered. With considering Eq. (4), the increment of the kinetic energy determines:

$$dE_k = [m(v_x w_x + v_y w_y)_C + \omega\varepsilon J_C]dt$$

Here, w_x, w_y—components of acceleration of the center of mass, ε—angular acceleration.

As generalized coordinates can be coordinates of the center of mass x_C, y_C and the angle φ,

$$dE_k = (mw_x dx + mw_y dy)_C + \varepsilon J_C d\varphi \quad (6)$$

or their speed

$$dE_k = (mv_x dv_x + mv_y dv_y)_C + \omega J_C d\omega$$

For centrifugal and Newtonian forces, Eq. (6) must be used. Thus, we get three generalized forces:

$$dE_k = Q_{kx}dx_C + Q_{ky}dy_C + Q_{k\varphi}d\varphi \quad (7)$$

each of which depends on the mass (inertia) properties of the body, as well as linear or angular accelerations:

$$(Q_{kx})_C = F_x = m(w_x)_C$$
$$(Q_{ky})_C = F_y = m(w_y)_C$$
$$(Q_{k\phi})_C = M_C = \varepsilon J_C \quad (8)$$

The first two (Newtonian) force Q_{kx} and Q_{ky} formally coincide with the determined by Newton's law and they determine the increment of the kinetic energy E_k of a rigid body, progressive movement of a rigid body, on the increment of coordinates $(dx)_C$, $(dy)_C$, the center of mass of the body with respect to the proposed fixed coordinate system of the observer. The third force $Q_{k\varphi}$ defines the increment of the rotational kinetic energy in Eq. (3) on increment of the rotation angle of the body. Only the simultaneous use of all three forces and the corresponding kinematic of coordinates allow you to find the increment of the invariant (scalar) the characteristics of body motion-kinetic energy E_k.

From Eq. (8), it follows that the relation $F = ma$ may be regarded as the definition of the generalized forces to calculate the increments of the kinetic energy of the body on increments of coordinate the center of mass relative to the beginning of the coordinate system of the observer.

To understand the meaning of the centrifugal forces, the submission of kinetic energy by Eq. (4) is considered when the pole P is aligned with the ICV:

$$E_k = 0.5\omega^2 J_{ICV} = 0.5\omega^2 (J_C + m\rho_C^2) \quad (9)$$

where ρ_C—the distance between the center of mass C and ICV, whose coordinates are determined by Eq. (5). Considering that the mass m and moment of inertia about the central axis J_C remain unchanged, the increase of two parameters: the angle φ and the distance ρ_C should be taken into account in the increase of energy:

$$dE_k = \varepsilon(J_C + m\rho_C^2)d\varphi + m\omega^2 \rho_C d\rho_C \quad (10)$$

The result is two generalized forces: torque with respect to ICV $Q_{k\varphi}$ (Nm) and force $Q_{\rho C}$ with dimension (N):

$$dE_k = Q_{k\varphi}d\varphi + Q_{\rho C}d\rho_C$$
$$Q_{k\phi} = (J_C + m\rho_C^2)\varepsilon = J_{ICV}\varepsilon \quad (11)$$
$$Q_{\rho C} = m\omega^2 \rho_C = mv_C^2 / \rho_C$$

Generalized force $Q_{\rho C}$ is identical in magnitude and direction [2, 8] with the generally definition of centrifugal force. Force $Q_{\rho C}$ as forces Q_{kx}, Q_{ky} in Eq. (8), determines the increment of the kinetic energy, the translational movement of the body, but on increments of distance between C and ICV. Moment $Q_{k\varphi}$ with the dimension (Nm) is also different from

that used in the system Eq. (8) due to the transition from the central axis to the axis passing through the ICV.

Since the curvature of the trajectory of the center of mass C may occur in any case of plane motion, calculating the change in kinetic energy can always be used either in Newtonian or centrifugal force (together with the corresponding moments $Q_{k\varphi}$). But the force Newton is more subjective, compared with centrifugal force, so they characterize the increment of the kinetic energy on increment of distance from the center of mass to coordinate system of observer.

Centrifugal force is characterized by the increase of the kinetic energy on the increment of the distance between the centers of mass and ICV of a moving body that varies over time but does not depend on the choice of the coordinate system of the observer.

The integral for the volume centrifugal and Newtonian forces can be determined in all cases; they are characterized by increase of kinetic energy body for incremental distance either between the center of mass of the body and the ICV, or between the center of gravity and the coordinate system of the observer. It is generally considered acceleration in the coordinate system of the observer and associated with Newtonian force.

3. Results and Discussion

Centrifugal force is usually associated with a sense of human by motion with a fairly high speed on curved track sections [1, 2]. Energy and kinematical conditions for certain movements are considered.

When driving in a straight line, if the x-axis is directed along the direction of motion, from Eq. (3), we get

$$E_k = 0.5mv_x^2$$

Since the mass does not change with the arrival of energy,

$$dE_k = mv_x w_x dt > 0$$

is the only one possible option: signs of acceleration and speed are the same, the speed increases and the observer feels this acceleration of motion.

If the mechanical system sends a power in external environment ($dE_k < 0$), the velocity and acceleration should be of opposite signs, the inhibition of the system. It is also possible when the kinetic energy of the body does not change because of $w_x = 0$,

$$dE_k = mv_x w_x dt = 0$$

but the movement continues at a constant velocity ($v_x = const$).

When rotational movement of a fixed axis is coincident with the center of mass, the distance between C and the ICV is 0 (the point C and the ICV are the same) for the kinetic energy from Eq. (9), we obtain

$$E_k = 0.5\omega^2 J_{ICV} = 0.5\omega^2 J_C$$

Since the ICV is stationary in this case, it can be taken as the origin of the coordinate system of the observer. Logically in this embodiment, Newtonian and centrifugal forces must be the same. But since the points C and ICV are the same, the question of the existence of these forces is not meaningless. The increment of kinetic energy defines a generalized force-torque:

$$dE_k = Q_{k\varphi} d\varphi, \quad Q_{k\phi} = J_C \varepsilon$$

Moment of inertia J_C depends on the size and shape of the body. At constant moment of inertia J_C, exchange energy with the environment leads to the angular acceleration, and therefore, a change in the angular velocity with increase (the signs of ε and ω are the same), or decrease of the rotation (the signs of ε and ω do not coincide) are possible.

But if the system is rotating around its central fixed axis (passed through the center of mass) at a constant angular velocity, the kinetic energy of the body does not change; and if there is no friction, the motion may continue indefinitely.

If the axis of rotation passes through the center of mass of the body (flying saucer), the pole may be aligned with the center of mass $\rho_C = 0$. The positive power increases or the velocity of the center of mass, or the angular velocity of rotational motion.

The rotational motion around a fixed axis that is not coincident with the center of mass, permits variation

of the distance between C and the ICV, which leads to a change in moment of inertia J_{ICV}. The change of energy in Eq. (10) determines two components: the distance between C and ICV, and the angle of rotation of the body. Increasing power ($dE_k > 0$) can be either increasing the angular velocity (in case of identical signs of the acceleration and the increment of the angle) with a constant radius of curvature of the trajectory of the center of mass ρ, or increasing the radius of curvature ρ and the moment of inertia of the body J_{ICV} at a constant angular velocity. Simultaneous change of two kinematic parameters is possible. Negative power lead forces can slow rotation or reduce the moment of inertia when the center of mass approaches the center of rotation. The continuation of the rotation with a constant angular velocity at a constant distance between the center of mass and axis of rotation is also possible.

If we assume that the motion of the system at fixed energy is inertia movement (similar to Newton's law 1), then the centrifugal forces can be regarded as forces of inertia.

In the general case of plane-parallel movement, the body rotates around the axis with the pole P, which does not coincide with the center of mass. For any time, you can use Eq. (3), which allows you to use six generalized forces. However, it is more convenient to combine the origin with ICV and determine the increment of kinetic energy due to the force in Eq. (11). The displacement of the mass center from the axis of rotation is perceived as centrifugal force.

For any plane-parallel movement with a curved trajectory, you can select the coordinate system of the observer when the axis is directed along the tangent to the trajectory (y) and orthogonal (x) to it. In this case, the tangential velocity will be directed along y-axis. However, the acceleration can be tangential w^τ (along the y-axis) and the normal w^n (along the x-axis in the direction of concavity).

Overall, as in any other coordinate system of observer, we obtain two projections of the acceleration; for the increment of the kinetic energy in this system of coordinates with pole at the center of mass, we obtain by analogy with Eqs. (7) and (8):

$$dE_k = (F_x)_C dx_C + (F_y)_C dy_C + M_C d\phi =$$
$$(mw^n)_C dx_C + (mw^\tau)_C dy_C + M_C d\varphi$$
$$F_x = mw_C^n = mv_C^2 / \rho_C$$
$$F_y = mw^\tau = mdv_C / dt$$
$$M_C = J_C \varepsilon$$

where ρ_C—radius curvature of the trajectory of the center of mass at the relevant point in time, F_x—the centripetal force, F_y—the path tangent (accelerate or decelerate) the power, M_C—torque.

Using the terminology of classical mechanics, based on the above considerations, centripetal force can be regarded as a special case of Newtonian forces. The point of application of force projections F_x and F_y should be the center of mass since the increment of the coordinate of this point is used to calculate the increment of the kinetic energy.

In other words, due to the special choice of the coordinate system, we can assume that in the center of mass of a rigid body there are three forces: the tangential force (in relation to the direction of motion), which determines the change of the absolute value of the velocity of the center of mass, and two symmetrical orthogonal forces: centrifugal, directed from the center of rotation and centripetal—towards the center of rotation. We can assume that these two forces hold the center of mass of the body on this trajectory.

Separately, we note the most frequently used division of the kinetic energy for translational and rotational components. This is wrong; they are not invariant and are modified after the transition to the new pole. Only the total kinetic energy of a body is invariant as the sum in the right-hand side of Eq. (3). The value of E_k, along with other options, can be obtained using centrifugal force, combining pole with ICV.

If the distance between the instantaneous axis of rotation and the center of mass does not change, the

centrifugal forces are moving into the category of passive, which produce power only when changing kinematic relations that maintain the distance between the instantaneous axis of rotation and the center of mass. In the category of such forces, forces orthogonal guide the translational motion may also be included, as well as appearing in the axes of links hinged-lever mechanisms [9-11].

The above considerations make it possible to reduce energy costs in the projected machines and mechanisms, using options with minimal changes of kinetic energy, approximating the real movement towards the ideal: translational motion in a straight line at a constant speed, rotational movement of the body with a constant angular velocity about its own fixed central axis or axis extending through the fixed ICV, which does not coincide with the movable center of mass, with a constant distance from the center of mass to ICV (generator Searle).

Information about centrifugal forces and the provisions of ICV in modeling the operation of the lever mechanisms on computers allows us to offer new methods of reducing power and energy consumption. For this purpose, it is desirable that the centers of mass of the machine parts were close to the ICV, when the generalized forces along the axes of the hinges or on the drive shaft have a maximum value [11].

4. Conclusions

The above can be considered as the basis of energy models mechanics with three basic concepts: space, time and energy. All the main provisions of modern mechanics, including Newton's laws for a point mass, the hypothesis of statics, the laws of Hooke and Hugoniot [1] and others, can be obtained as private cases from the law of conservation of energy. The interaction of bodies in motion is reduced to the transmission of energy and its transformation from one form to another [8].

By definition, in the energy model, the mechanics of force is designed to calculate changes in the generalized scalar function of movement and use it in a generalized law of conservation of energy.

For characteristics to change the kinetic energy, like any other, different kinematic coordinates can be used, and hence, different combinations of the generalized forces.

The forces of Newton and the centrifugal force with the corresponding torques are equivalent in terms of change of kinetic energy, and differ only in the choice of coordinate system and kinematics parameters: a fixed coordinate system for Newtonian forces and the moving coordinate system with origin at ICV for centrifugal forces.

Reducing the cost of energy to the movement is possible by the convergence of the trajectory of the center of mass and the ICV, including by changing the position of the center of mass of parts of the mechanisms provided for dynamic balancing of mechanisms [11].

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